تم الرفع بواسطة
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جبر خطي
هندسة كهربائية

Linear

27/40

Palestine Technical University
Department of Mathematics
Linear Algebra
First Exam
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1 Hour

True / False. (1..0 points)

1. T $(A+B)^{-1} = B^{-1} + A^{-1}$.
2. T If A *is a* 3×3 matrix, then there are two triangular matrices that factorizes A.
3. F If A *is a* 3×5 and the system $Ax = b$ is consistent, then it has a unique solution.
4. T Elementary row operations never change the solution set to a linear system.
5. F The reduced row echelon form of a matrix is unique.
6. F If b is a linear combination of vectors a_1, a_2, and a_3 then the constants used in the linear combination are unique.
7. T If A, B *and* C *are* $n \times n$ nonsingular matrices then the system $(ABC)x = 0$ has only the trivial solution.
8. F The second row of AB is the second row of A multiplied on the right by the matrix B.
9. F If A is an invertible $n \times n$ matrix then the equation $Ax = b$ has exactly one solution for each vector b in R^n.
10. F The cofactor expansion of a matrix going down a column is the negative of the cofactor expansion of the same matrix going across a row.

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2. If A is an 4×4 matrix, and the homogenous system $Ax = 0$ has only the trivial solution. Show that if $C = \alpha(A^T A)$, $\alpha \neq 0$, then C is nonsingular, then show that $de\,t(C) > 0$.

$Ax = 0 \Rightarrow A$ is nonsingular.

$c = \alpha(A^T A) \Rightarrow C$ is scalar multiply by a nonsingular matrix so C is nonsingular

$\cancel{\det A^T = \det A}$, $\det C = \alpha \det A^T \det A \Rightarrow \det C = \alpha \det^2 A$ why?

α is positive because negativ sign of α ~~change~~ make c singular $C = -\det A$!

$\cancel{\therefore \det C = \underbrace{\alpha}_{\text{Positive}} \underbrace{\det^2 A}_{\text{Positive}}}$ $\therefore \det C$ is Positive $\Rightarrow > 0$

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3. Consider the matrix $A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 2 & 2 \\ 5 & 0 & 1 \end{bmatrix}$ (12 points)

i. Find *adjoint(A)* .

$$Adj(A) = \begin{bmatrix} \begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & 2 \\ 5 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 5 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 0 \\ 5 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 5 & 0 \end{vmatrix} \\ \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 0 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} 2 & 11 & -10 \\ -1 & 3 & 5 \\ 2 & -6 & 7 \end{bmatrix}^T$$

$$\therefore Adj(A) = \begin{bmatrix} 2 & 11 & -10 \\ -1 & 3 & 5 \\ 2 & -6 & 7 \end{bmatrix}$$

ii. Use cofactor expansion to find *det(A)*.

~~$det(A) = det(Adj(A))$~~

$$det(A) = 3\begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} - 1 * \begin{vmatrix} -1 & 2 \\ 5 & 1 \end{vmatrix} + 0 * \begin{vmatrix} -1 & 2 \\ 5 & 0 \end{vmatrix} = (3*2) + 11 = 6 + 11 = \underline{\underline{17}}$$

expansion about Row I

iii. Use your answers in parts (i) and (ii) to find the inverse of *A*.

$$A^{-1} = \frac{1}{det(A)} * Adj\,A \quad , \quad detA = 17 \quad , \quad Adj\,A = \begin{bmatrix} 2 & 11 & -10 \\ -1 & 3 & 5 \\ 2 & -6 & 7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{17} * \begin{bmatrix} 2 & 11 & -10 \\ -1 & 3 & 5 \\ 2 & -6 & 7 \end{bmatrix} = \begin{bmatrix} \frac{2}{17} & \frac{11}{17} & \frac{-10}{17} \\ \frac{-1}{17} & \frac{3}{17} & \frac{5}{17} \\ \frac{2}{17} & \frac{-6}{17} & \frac{7}{17} \end{bmatrix} \Rightarrow \therefore A^{-1} =$$

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4. Consider the matrix $A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 3 & 5 & 0 & 2 \\ -4 & 2 & 1 & 2 \\ 1 & 1 & 4 & 5 \end{bmatrix}$ (12 points)

i. Use row reduction to find the determinant of A. Once the matrix is in upper-triangular form (but not before) you may finish the determinant.

change to upper tringlur :-

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 3 & 5 & 0 & 2 \\ -4 & 2 & 1 & 2 \\ 1 & 1 & 4 & 5 \end{bmatrix} \xrightarrow{\substack{R_1 = R_4 \\ R_4 = R_1}} \begin{bmatrix} 1 & 1 & 4 & 5 \\ 3 & 5 & 0 & 2 \\ -4 & 2 & 1 & 2 \\ 2 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{\substack{R_4 = R_4 - 2R_1 \\ R_2 = R_2 - 3R_1 \\ R_3 = R_3 + 4R_1}} \begin{bmatrix} 1 & 1 & 4 & 5 \\ 0 & 2 & -12 & -13 \\ 0 & 6 & 17 & 22 \\ 0 & -2 & -7 & -10 \end{bmatrix} \xrightarrow{\substack{R_3 = R_3 - 3R_2 \\ R_4 = R_4 + R_2}}$$

$$\begin{bmatrix} 1 & 1 & 4 & 5 \\ 0 & 2 & -12 & -13 \\ 0 & 0 & 53 & 61 \\ 0 & 0 & -19 & -23 \end{bmatrix} \xrightarrow{R_4 = R_4 + \frac{19}{53} R_3} \begin{bmatrix} 1 & 1 & 4 & 5 \\ 0 & 2 & -12 & -13 \\ 0 & 0 & 53 & 61 \\ 0 & 0 & 0 & (61 \times \frac{19}{53}) - 23 \end{bmatrix}$$

$53 \overline{)1159}$ = 21, 106, 99, 53, 46

61 × 19 = 549 + 610 = 1159

$\therefore 21.\frac{46}{53} - 23$

$21.84 - 23$

$= -1.15$

$$A = \begin{bmatrix} 1 & 1 & 4 & 5 \\ 0 & 2 & -12 & -13 \\ 0 & 0 & 53 & 61 \\ 0 & 0 & 0 & -1.15 \end{bmatrix}$$

$\therefore$ det (A) = multiple of elements on diagonal.

$= 1 * 2 * 53 * -1.15 = 106 * -1.15 = -121.5$

$\therefore \det(A) = -121.5$

ii. Determine whether A is singular or nonsingular? Why?

A is nonsingular because $\det(A) \neq 0$

iii. Use the matrix obtained in (i) to solve the system $Ax = [11 \quad 2 \quad 0 \quad 0]^T$

$$\left[\begin{array}{cccc|c} 1 & 1 & 4 & 5 & 11 \\ 0 & 2 & -12 & -13 & 2 \\ 0 & 0 & 53 & 61 & 0 \\ 0 & 0 & 0 & -1.15 & 0 \end{array}\right]$$

$\Rightarrow -1.15 X_4 = 0 \quad \therefore X_4 = 0$

$53 X_3 + 0 = 0 \quad \therefore X_3 = 0$

$2X_2 - 0 - 0 = 2 \quad \therefore X_2 = 1$

$X_1 + 1 + 0 + 0 = 11 \quad \therefore X_1 = 10$

$\therefore X = \begin{bmatrix} 10 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

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